

T - Network Phase Shift Network Design

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LC impedance matching networks are used extensively in MMIC Phase Shifters. Phase Shifters are used in phase array antennas, steerable communication links, and high linearity amplifier cancellation loops. The T-networks are used because each network can be designed for a specific phase shift.

This white paper's purpose is to help me connect concepts and clarify them by writing down the ideas. If you are reading this, then hopefully you will gain a better understanding designing a T-matching network for a specific phase shift.

A sine wave has a leading phase angle (advance) when the positive slope passes through the horizontal axis before $t = 0$. **Figure 1** shows a leading (advance) sine wave.

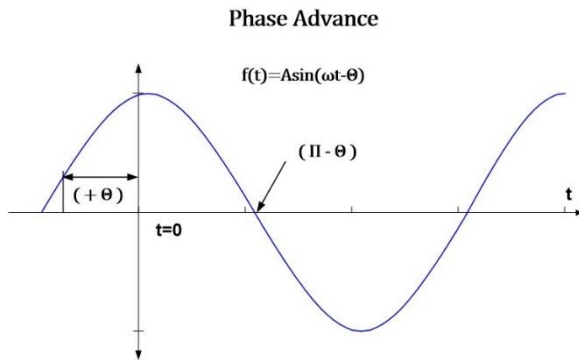


Figure 1: Phase Advance Sine wave

Figure 2 shows that a phase delay is a CCW rotation in phasor form.

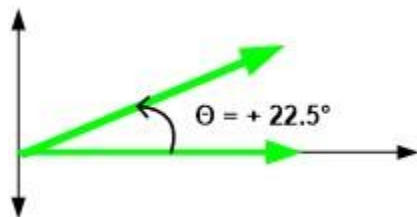


Figure 2: Phasor form Representation

In the subsequent phasor diagrams, dotted lines represent the current phasors, and solid lines represent the voltage phasors. This is done so the reader can easily view the 90° between current and voltage phasors in each reactive element. In an inductor, voltage leads current by 90° . But, current leads voltage by 90° in a capacitor.

A T-matching network will be designed to match a 100Ω source resistance to a 50Ω load resistance from 2.5 to 4.0 GHz. The signal will undergo $+22.5^\circ$ phase shift, θ , through the network.

Figure 3 shows all the voltages and currents labeled. The unknown ideal reactive elements are labeled X_1 , X_2 , and X_3 .

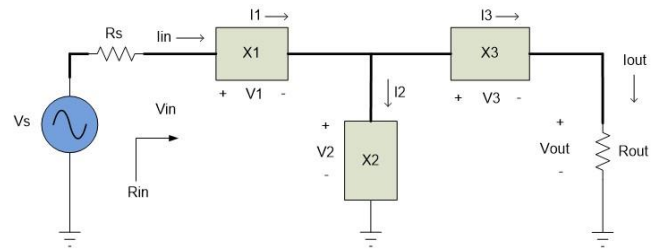


Figure 3: T - Matching Network

Impedance matching provides maximum power transfer from the source to the load at specific frequency and a good return loss over specified bandwidth. The T-network is similar to an ideal transformer. Each provide maximum power transfer, and they have the same terminal voltage and current characteristics. Since the T network's reactive elements are ideal, $P_{in} = P_{out}$. The design starts by setting P_{out} . Choosing $I_{out} = 1A$ so that the $P_{out} = R_{out}$ results in $V_{out} = R_{out}$. This simplifies the V_{in} , I_{in} , V_{out} , and I_{out} calculations, but the designer can start with any I_{out} value they desire.

$$\frac{V_{out}}{V_{in}} = \sqrt{a} \quad (1)$$

$$\frac{I_{in}}{I_{out}} = \sqrt{a} \quad (2)$$

$$a = \frac{R_{out}}{R_{in}} \quad (3)$$

Table 1: Ideal Transformer Voltage and Current EQNs

Where a is defined as the impedance transformation ratio. Use (1), (2), and (3) to calculate V_{in} and I_{in} .

Table 2 summarizes the input and output currents and voltages.

$V_{out} = 50 \text{ V}$	$I_{out} = 1 \text{ A}$	$P_{out} = 50 \text{ W}$
$V_{in} = 50\sqrt{2}$	$I_{in} = \frac{\sqrt{2}}{2}$	$P_{in} = 50 \text{ W}$

Table 2: I/O Voltages and Currents

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In the illustrations that follow, the voltage and current phasors use different scales. No relationship exists between either scale. The current phasor lengths are drawn at the correct ratio as is voltage phasor length. Microsoft 2003 Visio was used to draw the phasor diagrams.

The voltage phasor diagram is drawn using the head to tail method also called the triangle method. The arrow points in a potential increase, from the negative terminal to positive terminal on the element. The voltage phasors will always form a right triangle.

The current phasor diagrams are drawn using the head to tail method flowing out of the node. The hypotenuse does not need to be the sum of square of the sides.

Figure 4 illustrates the input current and voltage phasors; and the output current and voltage phasors.

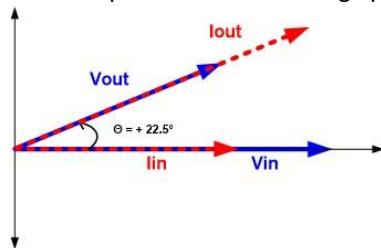


Figure 4: Input and Output Phasors

Using **Figure 3** and Kirchhoff's laws:

$$V_2 = V_3 + V_{out} \tag{4}$$

$$I_{in} = I_2 + I_{out} \tag{5}$$

$$V_{in} = V_1 + V_2 \tag{6}$$

$$I_{in} = I_1 \tag{7}$$

$$I_{out} = I_3 \tag{8}$$

Using equation (5), **Figure 5** shows the current phasor diagram.

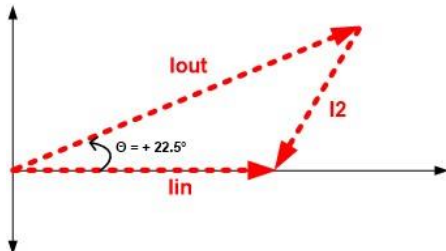


Figure 5: Current Phasor Diagram

Using the Law of Cosines, to calculate I₂:

$$I_2 = \sqrt{(I_{in})^2 + (I_{out})^2 - 2(I_{in})(I_{out}) \cos(\theta)} \tag{9}$$

$$I_2 = \sqrt{\left(\frac{\sqrt{2}}{2}\right)^2 + (1)^2 - 2\left(\frac{\sqrt{2}}{2}\right)(1) \cos(22.5)}$$

$$I_2 = 0.4398 \text{ A}$$

Each reactive element's voltage phasor will be drawn independently. The V₂ phasor will be drawn first. V₂'s position is known because of the boundary conditions that the output voltage lead the input voltage by 22.5° and I₂ is perpendicular to V₂. But, V₂'s direction is unknown until we draw V₃. **Figure 6** illustrates the V₂ phasor.

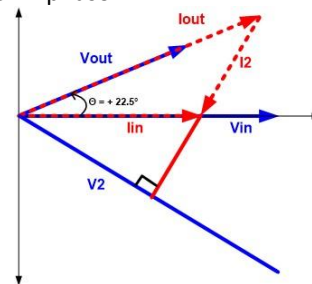


Figure 6: V₂ Phasor Diagram

Using V₃ is perpendicular to I_{out} and (4) to determine V₂ direction, **Figure 7** shows the V₂ and V₃ phasor diagram.

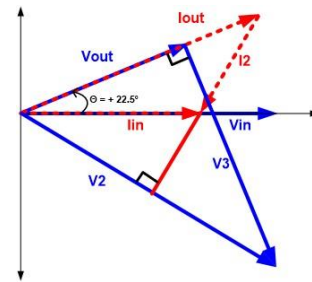


Figure 7: V₂ & V₃ Phasor Diagrams

Using (6) and that I_{in} is perpendicular to V₁, **Figure 8** illustrates the V₁ phasor.

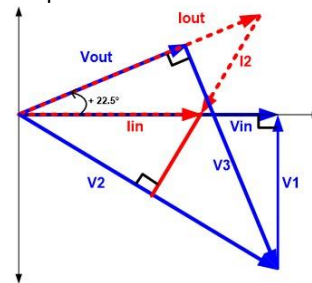


Figure 8: T-Network Phasor Diagram

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Using **Figure 8**, each reactive element type can be determined. X3 is a capacitor because Iout is leading V3. But, X2 is an inductor because V2 is leading I2. Similarly, X1 is an inductor because V1 is leading Iin.

Figure 9 illustrates the T-network with the reactive elements.

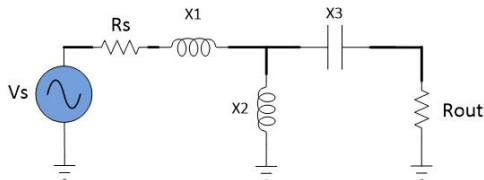


Figure 9: T-Network with reactive elements

Figure 10 reveals how to solve for current phasors. Let's define two angles in **Figure 10**. ϕ is the angle between I2 and Iout. α is the angle between V2 and Vout.

Calculate ϕ . Next, subtract ϕ from 90° to find α . Once α is known, use the trig functions to calculate V3, V2, and V1.

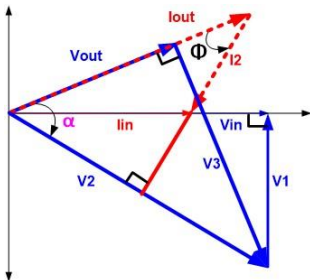


Figure 10: Current Phasor Calculation Diagram

Using the Law of Cosines, to calculate Φ :

$$\Phi = \cos^{-1} \left(\frac{(I_{in})^2 - (I_{out})^2 - (I_2)^2}{-2 * I_{out} * I_2} \right) \quad (10)$$

$$\Phi = \cos^{-1} \left(\frac{\left(\frac{\sqrt{2}}{2}\right)^2 - (1)^2 - (.4398)^2}{-2 * 1 * .4398} \right)$$

$$\Phi = 37.97^\circ$$

$$\alpha = 90^\circ - \Phi \quad (11)$$

$$\alpha = 52.03^\circ$$

Use Δ V2-V3-Vout and the tan formula to calculate V3.

$$\tan \alpha = \frac{V_3}{V_{out}} \quad (12)$$

$$V_3 = 50 * \tan(52.03)$$

$$V_3 = 64.068 \text{ V}$$

Use Δ V2-V3-Vout and the cos formula to calculate V2.

$$\cos \alpha = \frac{V_{out}}{V_2} \quad (13)$$

$$V_2 = \frac{50}{\cos(52.03^\circ)}$$

$$V_2 = 81.269 \text{ V}$$

Use Δ V1-V2-Vin and the sine formula to calculate I1.

$$\sin(\alpha - 22.5) = \frac{V_1}{V_2} \quad (14)$$

$$V_1 = 81.269 * \sin(52.03^\circ - 22.5^\circ)$$

$$V_1 = 40.057 \text{ V}$$

Table 3 summarizes each reactive element's voltage and current magnitude.

Vin=	$50\sqrt{2} \text{ V}$	Iin=	$\frac{\sqrt{2}}{2} \text{ A}$
Vout=	50 V	Iout=	1 A
V1=	40.057 V	I1=	$\frac{\sqrt{2}}{2} \text{ A}$
V2=	81.269 V	I2=	0.4398 A
V3=	64.068 V	I3=	1 A

Table 3: Voltage and Current Summary

Table 4 summarizes each element's reactance and component values at $f_0=3.25 \text{ GHz}$

X1=	56.649 Ω	L1=	2.774 nH
X2=	184.787 Ω	L2=	9.049 nH
X3=	64.068 Ω	C1=	0.764 pF

Table 4: +22.5° T-Network Reactance & Component Values

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Figure 11 shows the +22.5° T-network schematic.

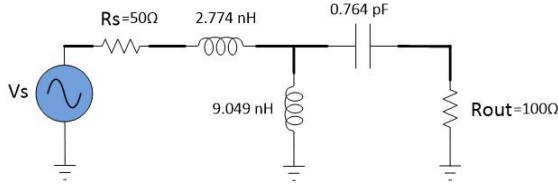
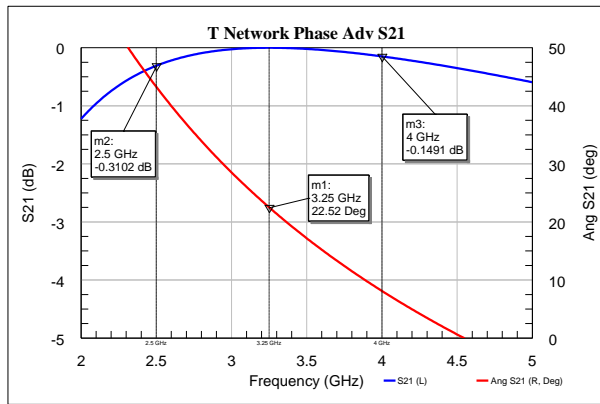
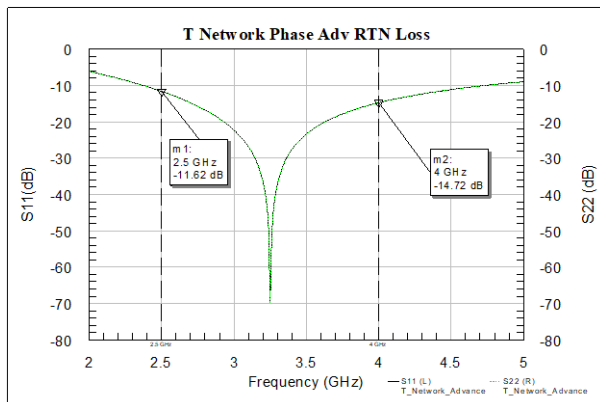


Figure 11: +22.5° T-Network Schematic

Using Microwave Office, Graphs 1 – 2 show s21, s21 phase, s11, and s22 responses.



Graph 1: +22.5° T-Network S21 Response



Graph 2: +22.5° T-Network RTN Loss Response

A -22.5° T-network can be designed using the +22.5° T-network reactance values, but using the opposite reactive element. Table 5 shows the new element values, and Figure 12 shows the new schematic.

X1=	56.649 Ω	C1=	0.864 pF
X2=	184.787 Ω	C2=	0.265 pF
X3=	64.068 Ω	L1=	3.137 nH

Table 5: -22.5° T-Network Reactance & Component Values

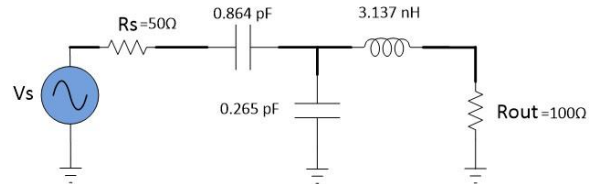
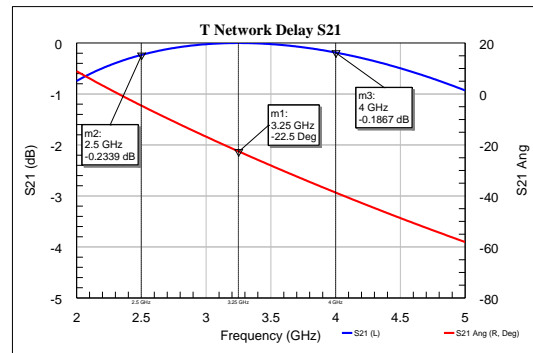
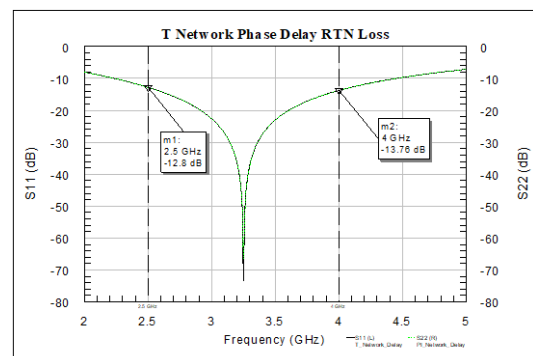


Figure 12: -22.5° T-Network Schematic

Graphs 3 – 4 show s21, s21 phase, s11, and s22 responses.



Graph 3: -22.5° T-Network S21 Response



Graph 4: -22.5° T-Network RTN Loss Response

Table 6 lists the π -Network phase delay design equations. The impedances X1, X2, and X3 are a function of Rout, Rin, and phase delay only.

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$$V_{out} = R_{out} \quad I_{out} = 1 \text{ A} \quad (15)$$

$$V_{in} = \sqrt{(R_{out} * R_{in})} \quad I_{in} = I_{out} * \sqrt{\frac{R_{out}}{R_{in}}} \quad (16)$$

$$I_2 = \sqrt{\left(\frac{R_{out}}{R_{in}}\right) + (1)^2 - 2 * \left(\frac{R_{out}}{R_{in}}\right)^{\frac{1}{2}} * \cos(\theta)} \quad (17)$$

$$\Phi = \cos^{-1}\left(\frac{\left(\frac{R_{out}}{R_{in}}\right) - (1)^2 - (I_2)^2}{-2 * 1 * I_2}\right) \quad (18)$$

$$\alpha = 90^\circ - \Phi \quad (19)$$

$$V_3 = R_{out} * \tan \alpha \quad (20)$$

$$V_2 = \frac{R_{out}}{\cos(\alpha)} \quad (21)$$

$$V_1 = \frac{R_{out} * \sin(\alpha - \theta)}{\cos(\alpha)} \quad (22)$$

$$X_1 = \frac{V_{in}}{I_1} \quad X_2 = \frac{V_2}{I_2} \quad X_3 = \frac{V_3}{I_3} \quad (23)$$

Table 6: π -Network Design Equations

Let's examine α in (19), (20), and (21). α will always be less than 90° . **Figure 13** shows α is an angle in right Δ ; therefore, I_{out} will always lead V_3 resulting in X_3 always being a capacitor. V_2 will always lead I_2 . Thus, X_2 will always be an inductor.

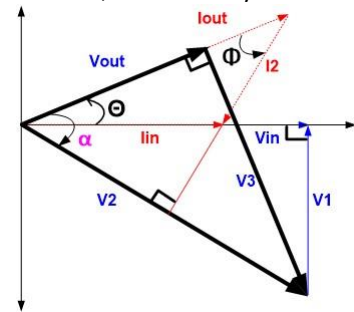


Figure 13: I_1 Reactance Changes Phasor Diagram

But, this is not the case with V_1 . **Figure 14** shows a 60° advance and **Figure 15** shows a 120° phase advance.

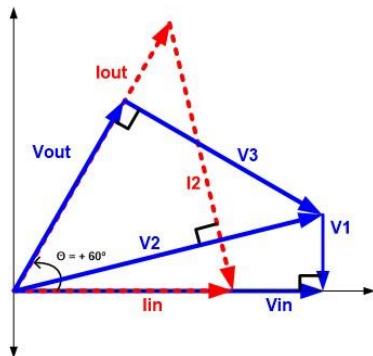


Figure 14: 60° Phase Advance Phasor Diagram

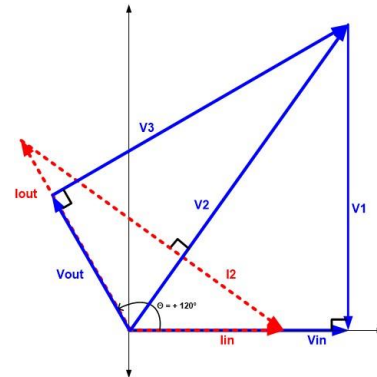


Figure 15: 120° Phase Advance Phasor Diagram

Figures 14 and 15 show I_3 leads V_{in} and the reactive element is now a capacitor. This occurs when $\sin(\Phi - \theta)$ in equation (22) is negative. To determine when this change occurs, a simple condition can be derived. Substituting (19) in the numerator of (22)

$$\sin(90^\circ - \Phi - \theta) \quad (24)$$

$$\sin[90 - (\Phi + \theta)] \quad (25)$$

We have two conditions. The first condition is:

$$\Phi + \theta = 90 \quad (26)$$

$$V_1 = \frac{R_{out} * \sin(90^\circ - 90^\circ)}{\cos(\Phi)}$$

$$V_1 = 0$$

$$X_1 = 0$$

When the α between I_2 and I_{out} plus the desired phase shift equals 90° , no X_1 element is required. The resulting matching network is a two element LC matching network is required.

The second condition is when

$$\Phi + \theta > 90 \quad (27)$$

The $\sin()$ becomes negative and V_1 has changed direction. The α between I_2 and I_{out} plus the desired phase shift is greater than 90° , use a capacitor for X_1 .

The phase shift when the reactive element changes depends on how close R_{out} and R_{in} are.

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Table 7 gives a couple of examples.

Rout	Rin	θ
200	50	60°
150	50	54.75°
100	50	45°
75	50	35.25°
50	50	0°

Table 7: Phase shift $\Phi + \theta = 90$; LC Matching Network

Any phase shift greater than θ results in X1 being a capacitor.

A few closing statements before presenting the final design equations.

1. Three specific topologies when designing a T-matching network for impedance matching and a specific phase shift depending on the $\angle \Phi$, the angle between I2 and Iout, and θ , the desired phase shift.

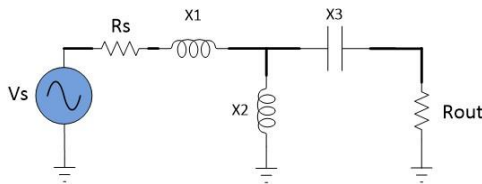


Figure 16: $\Phi + \theta < 90$

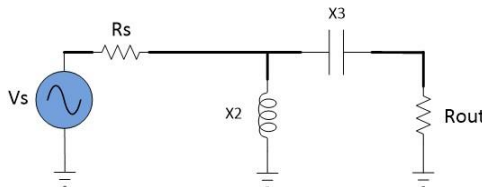


Figure 17: $\Phi + \theta = 90$

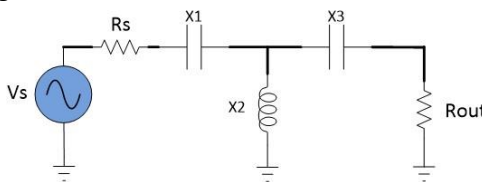


Figure 18: $\Phi + \theta > 90$

2. This white paper design used a higher source resistance than the load. When the load resistance is higher than the source, the designer can still use these equations. But,

the designer must reverse the source reactance, X1, and the load reactance, X3, in the Figures 16 - 18.

3. Analysis is with real resistance values for the source and load. Two approaches to handle reactive impedances. First, tune out the reactance, then use the Table 8 design equations. This will result in using more than three elements. Second, use the phasor methods in this white paper to determine the reactance values.
4. This white paper calculated each reactance in a phase advance network. If the designer desires a phase delay network, then use the same phase advance reactance. But, use opposite reactance element and solve for the new element value using the appropriate reactance equation.

$$V_{out} = R_{out} \quad I_{out} = 1 \text{ A} \quad (28)$$

$$V_{in} = \sqrt{(R_{out} * R_{in})} \quad I_{in} = I_{out} * \sqrt{\frac{R_{out}}{R_{in}}} \quad (29)$$

$$I_2 = \sqrt{\left(\frac{R_{out}}{R_{in}}\right) + (1)^2 - 2 * \left(\frac{R_{out}}{R_{in}}\right)^{\frac{1}{2}} * \cos(\theta)} \quad (30)$$

$$\Phi = \cos^{-1}\left(\frac{\left(\frac{R_{out}}{R_{in}}\right) - (1)^2 - (I_2)^2}{-2 * 1 * I_2}\right) \quad (31)$$

$$\alpha = 90^\circ - \Phi \quad (32)$$

$$I_3 = R_{out} * \tan \alpha \quad (33)$$

$$I_2 = \frac{R_{out}}{\cos(\alpha)} \quad (34)$$

$$I_1 = \frac{R_{out} * \sin(\alpha - \theta)}{\cos(\alpha)} \quad (35)$$

$$X_1 = +j \frac{V_{in}}{I_1} \quad \Phi + \theta < 90 \quad (36)$$

$$X_1 = 0 \quad \Phi + \theta = 90$$

$$X_1 = -j \frac{V_{in}}{I_1} \quad \Phi + \theta > 90$$

$$X_2 = +j \frac{V_2}{I_2} \quad (37)$$

$$X_3 = -j \frac{V_{out}}{I_{out}} \quad (38)$$

Table 8: T-Network Design Equations;