π - Network Phase Shift Network Design

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LC impedance matching networks are used extensively in MMIC Phase Shifters. Phase Shifters are used in phase array antennas, steerable communication links, and high linearity amplifier cancellation loops. The π-networks are used because each network can be designed for a specific phase shift.

This white paper's purpose is to help me connect concepts and clarify them by writing down the ideas. If you are reading this, then hopefully you will gain a better understanding designing a π-matching network for a specific phase shift.

A sine wave has a lagging phase angle (delay) when the positive slope passes through the horizontal axis after t = 0. **Figure 1** shows a lagging (delay) sine wave.

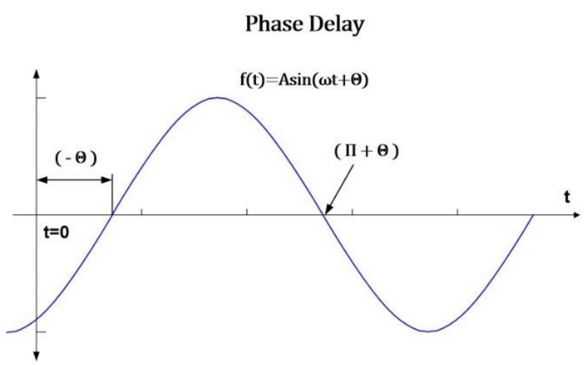

Figure 1: Phase Delay Sine wave

Figure 2 shows that a phase delay is a CW rotation in phasor form.

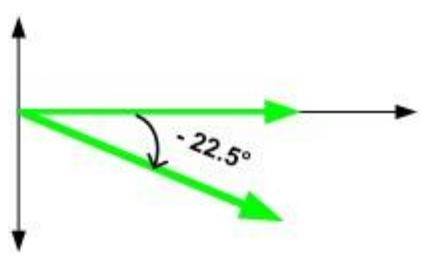

Figure 2: Phasor form Representation

In the subsequent phasor diagrams, dotted lines represent the current phasors, and solid lines represent the voltage phasors. This is done so the reader can easily view the 90° between current and voltage phasors in each reactive element. In an inductor, voltage leads current by 90°. But, current leads voltage by 90° in a capacitor.

A π-matching network will be designed to match a 50 Ω source resistance to a 100 Ω load resistance from 2.5 to 4.0 GHz. The signal will undergo -22.5° phase shift, Θ, through the network.

Figure 3 shows all the voltages and currents labeled. The unknown ideal reactive elements are labeled X_1, X_2, and X_3.

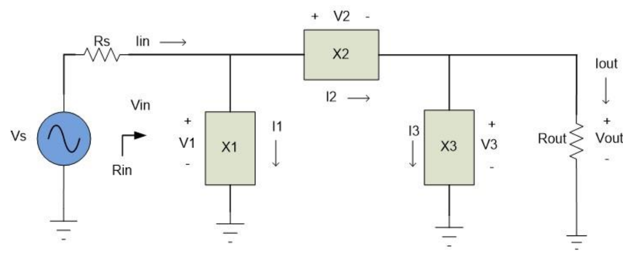

Figure 3: π - Matching Network

Impedance matching provides maximum power transfer from the source to the load at specific frequency and a good return loss over specified bandwidth. The π-network is similar to an ideal transformer. Each provide maximum power transfer, and they have the same terminal voltage and current characteristics. Since the π network's reactive elements are ideal, Pin = Pout. The design starts by setting Pout. Choosing Iout = 1A so that the Pout = Rout results in Vout = Rout. This simplifies the Vin, Iin, Vout, and Iout calculations, but the designer can start with any Iout value they desire.

$$\frac{\mathrm{Vout}}{\mathrm{Vin}} = \sqrt{a} \tag{1}$$

$$\frac{\mathrm{Iin}}{\mathrm{Iout}} = \sqrt{a} \tag{2}$$

$$a = \frac{\mathrm{Rout}}{\mathrm{Rin}} \tag{3}$$

Table 1: Ideal Transformer Voltage and Current EQNs

Where a is defined as the impedance transformation ratio. Use (1), (2), and (3) to calculate Vin and Iin. **Table 2** summarizes the input and output currents and voltages.

Vout = 100 V	Iout = 1 A	Pout = 100 W
Vin = $\frac{100}{\sqrt{2}}$	Iin = $\sqrt{2}$ *1	
Vin = 70.71 V	Iin = $\sqrt{2}$ A	Pin = 100 W

Table 2: I/O Voltages and Currents

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In the illustrations that follow, the voltage and current phasors use different scales. No relationship exists between either scale. The current phasor lengths are drawn at the correct ratio as is voltage phasor length. Microsoft 2003 Visio was used to draw the phasor diagrams.

The voltage phasor diagram is drawn using the head to tail method also called the triangle method. The arrow points in a potential increase, from the negative terminal to positive terminal on the element.

The current phasor diagrams are drawn using the head to tail method flowing out of the node. The hypotenuse does not need to be the sum of square of the sides. The current phasors will always form a right triangle.

Figure 4 illustrates the input current and voltage phasors; and the output current and voltage phasors.

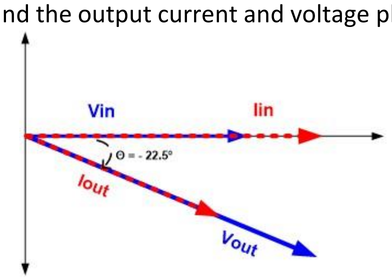

Figure 4: Input and Output Phasors

Using **Figure 3** and Kirchhoff's laws:

$$\mathrm{I2} = \mathrm{I3} + \mathrm{Iout} \quad (4)$$

$$\mathrm{Iin} = \mathrm{I1} + \mathrm{I2} \quad (5)$$

$$\mathrm{Vin} = \mathrm{V2} + \mathrm{Vout} \quad (6)$$

Vin = V1 (7)

Vout = V3 (8)

Using equation (6), **Figure 5** shows the voltage phasor diagram.

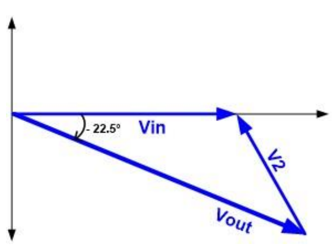

Figure 5: Voltage Phasor Diagram

Using the Law of Cosines, to calculate V2:

$$\mathrm{V2} = \sqrt{(\mathrm{Vin})^2 + (\mathrm{Vout})^2 - 2(\mathrm{Vin})(\mathrm{Vout})\cos(\Theta)} \quad (9)$$

$$\mathrm{V2} = \sqrt{(70.71)^2 + (100)^2 - 2(70.71)(100)\cos(22.5)}$$

V2 = 43.98 V

Each reactive element's current phasor will be drawn independently. The I2 phasor will be drawn first. I2's position is known because of the boundary conditions that the output voltage lag the input voltage by 22.5° and I2 is perpendicular to V2. But, I2's direction is unknown until we draw I3. **Figure 6** illustrates the I2 phasor.

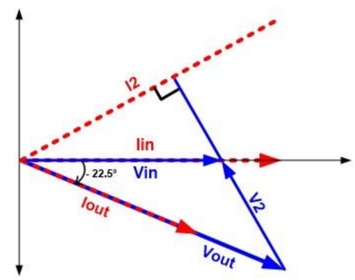

Figure 6: I2 Phasor Diagram

Using I3 is perpendicular to Vout and (4) to determine I2 direction, **Figure 7** shows the I2 and I3 phasor diagram.

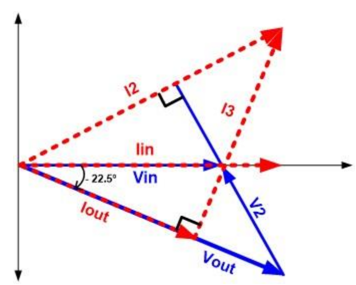

Figure 7: I2 & I3 Phasor Diagrams

Using (5) and that I1 is perpendicular to Vin, **Figure 8** illustrates the I1 phasor.

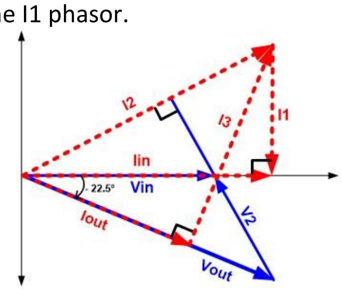

Figure 8: π-Network Phasor Diagram

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Using **Figure 8**, each reactive element type can be determined. X3 is a capacitor because I3 is leading Vout. But, X2 is an inductor because V2 is leading I2. Similarly, X1 is an inductor because V1 is leading I1.

Figure 9 illustrates the π-network with the reactive elements.

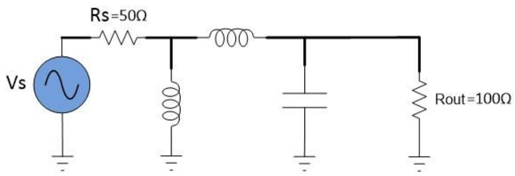

Figure 9: π-Network with reactive elements

Figure 10 reveals how to solve for current phasors. Let's define two angles in **Figure 10**. $\measuredangle\Phi$ is the angle between I2 and Iout. $\measuredangle\alpha$ is the angle between V2 and Vout.

Calculate $\measuredangle\alpha$. Next, subtract $\measuredangle\alpha$ from 90° to find $\measuredangle\Phi$. Once $\measuredangle\Phi$ is known, use the trig functions to calculate I3, I2, and I1.

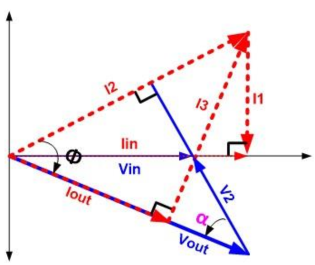

Figure 10: Current Phasor Calculation Diagram

Using the Law of Cosines, to calculate α:

$$\alpha = \cos^{-1}\left(\frac{(\text{Vin})^2 - (\text{Vout})^2 - (\text{V2})^2}{-2 * \text{Vout} * \text{V2}}\right) \quad (10)$$

$$\alpha = \cos^{-1}\left(\frac{70.71^2 - 100^2 - 43.98^2}{-2 * 100 * 43.98}\right)$$

α = 37.97°

Φ = 90° - α (11)

Φ = 52.03°

Use Δ I2-I3-Iout and the tan formula to calculate I3.

$$\tan\Phi = \frac{\text{I3}}{\text{Iout}} \quad (12)$$

$$\text{I3} = 1 * \tan(52.03)$$

I3 = 1.281 A

Use Δ I2-I3-Iout and the cos formula to calculate I2.

$$\cos\Phi = \frac{\text{Iout}}{\text{I2}} \quad (13)$$

$$\text{I2} = \frac{1}{\cos(52.03°)}$$

I2 = 1.625 A

Use Δ I1-I2-Iin and the sine formula to calculate I1.

$$\sin(\Phi - 22.5) = \frac{I1}{I2} \quad (14)$$

$$\text{I1} = 1.625 * \sin(52.03° - 22.5°)$$

I1 = 0.801 A

Table 3 summarizes each reactive element's voltage and current magnitude.

Vin=	70.7 V	Iin=	$\sqrt{2}$ A
Vout=	100 V	Iout=	1 A
V1=	70.71 V	I1=	0.801 A
V2=	43.98 V	I2=	1.625 A
V3=	100 V	I3=	1.281 A

Table 3: Voltage and Current Summary

Table 4 summarizes each element's reactance and component values at f_o=3.25 GHz

X1=	88.277 Ω	L1=	4.323 nH
X2=	27.065 Ω	L2=	1.325 nH
X3=	78.064 Ω	C1=	0.627 pF

Table 4: -22.5° π-Network Reactance & Component Values

Figure 11 shows the -22.5° π-network schematic.

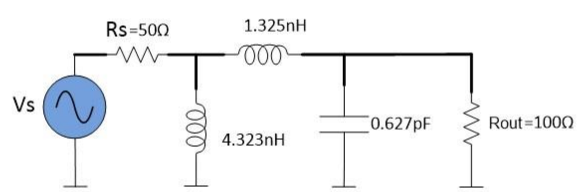

Figure 11: -22.5° π-Network Schematic

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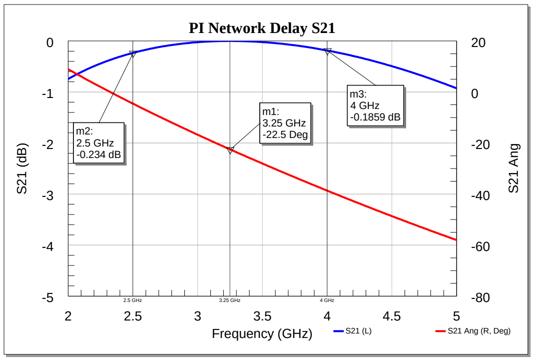
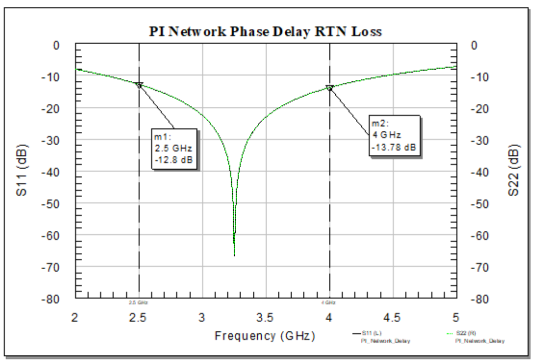
Using Microwave Office, **Graphs 1 – 2** show s21, s21 phase, s11, and s22 responses.



Graph 1: -22.5° π-Network S21 Response



Graph 2: -22.5°π-Network RTN Loss Response

A +22.5° π-network can be designed using the -22.5° π-network reactance values, but using the opposite reactive element. **Table 5** shows the new element values, and **Figure 12** shows the new schematic.

X1=	88.277 Ω	C1=	0.555 pF
X2=	27.065 Ω	C2=	1.809 pF
X3=	78.064 Ω	L1=	3.823 nH

Table 5: +22.5° π-Network Reactance & Component Values

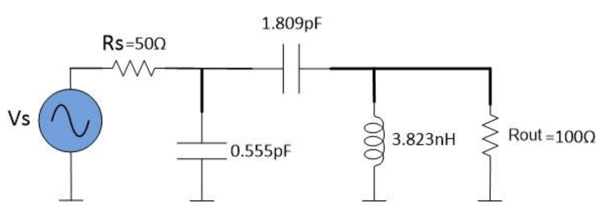
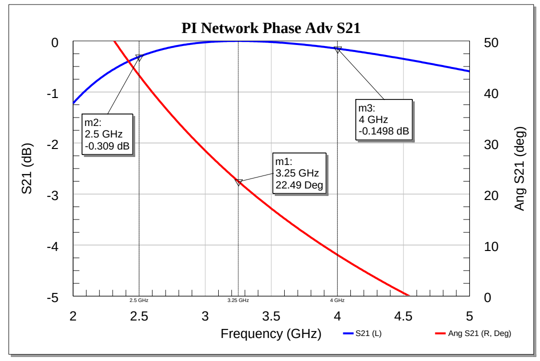
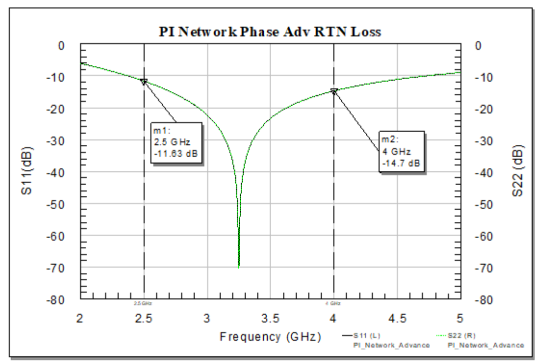


Figure 12: +22.5° π-Network Schematic

Graphs 3 – 4 show s21, s21 phase, s11, and s22 responses.



Graph 3: +22.5° π-Network S21 Response



Graph 4: +22.5° π-Network RTN Loss Response

Table 6 lists the π-Network phase delay design equations. The impedances X1, X2, and X3 are a function of Rout, Rin, and phase delay only.

$$Vout = Rout \qquad Iout = 1\ A \tag{15}$$

$$Vin = \sqrt{(Rout * Rin)} \qquad Iin = Iout * \sqrt{\frac{Rout}{Rin}} \tag{16}$$

$$V2 = \sqrt{(Rout * Rin) + (Rout)^2 - 2(Rout)^{\frac{3}{2}} * (Rin)^{\frac{1}{2}} * \cos(\Theta)} \tag{17}$$

$$\alpha = \cos^{-1}\left(\frac{(Rout * Rin) - (Rout)^2 - (V2)^2}{-2 * (Rout) * (V2)}\right) \tag{18}$$

$$\Phi = 90° - \alpha \tag{19}$$

$$I3 = Iout * \tan\Phi \tag{20}$$

$$I2 = \frac{Iout}{\cos(\Phi)} \tag{21}$$

$$I1 = \frac{\sin(\Phi - \Theta)}{\cos(\Phi)} \tag{22}$$

$$X1 = \frac{Vin}{I1} \qquad X2 = \frac{V2}{I2} \qquad X3 = \frac{Vout}{Iout} \tag{23}$$

Table 6: π-Network Design Equations

Let's examine $\measuredangle\Phi$ in (19), (20), and (21). $\measuredangle\Phi$ will always be less than 90°. **Figure 13** shows $\measuredangle\Phi$ is an angle in right Δ; therefore, I3 will always lead V3 resulting in X3 always being a capacitor. Similarly, V2 will always lead I2. Thus, X2 will always be an inductor.

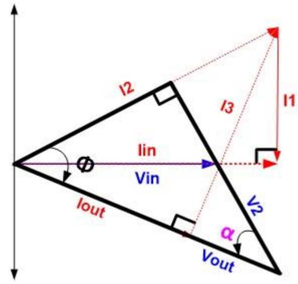


Figure 13: I1 Reactance Changes Phasor Diagram

But, this is not the case with I1. **Figure 14** shows a 60° delay and **Figure 15** shows a 120° phase delay.

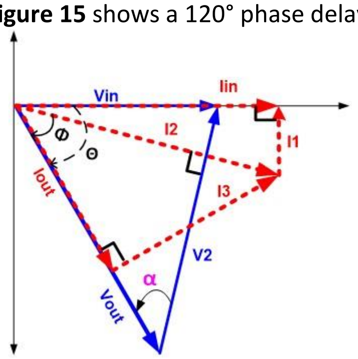


Figure 14: 60° Phase Delay Phasor Diagram

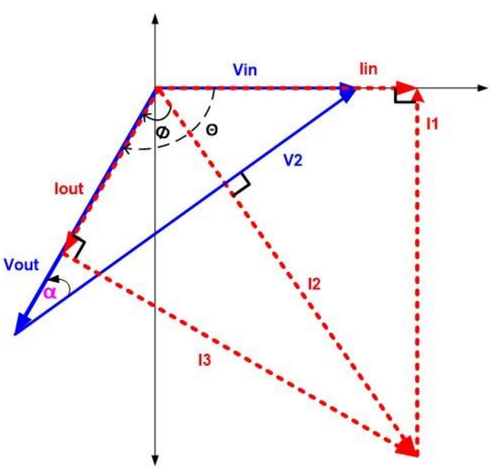


Figure 15: 120° Phase Delay Phasor Diagram

Figures 14 and 15 show I3 leads Vin and the reactive element is now a capacitor. This occurs when $\sin(\Phi - \Theta)$ in equation (22) is negative. To determine when this change occurs, a simple condition can be derived. Substituting (19) in the numerator of (22)

$$\sin(90° - \alpha - \Theta) \tag{24}$$

$$\sin[90 - (\alpha + \Theta)] \tag{25}$$

We have two conditions. The first condition is:

$$\alpha + \Theta = 90 \tag{26}$$

$$I1 = \frac{\sin(90° - 90°)}{\cos(\Phi)}$$

$$I1 = 0$$

X1 = ∞

When the $\measuredangle$ between V2 and Vout plus the desired phase shift equals 90°, no X1 element is required. The resulting matching network is a two element LC matching network is required.

The second condition is when

$$\alpha + \Theta > 90 \tag{27}$$

The sin () becomes negative and I3 has changed direction. The $\measuredangle$ between V2 and Vout plus the desired phase shift is greater than 90°, use a capacitor for X1.

The phase shift when the reactive element changes depends on how close Rout and Rin are. **Table 7** gives a couple of examples.

Rout	Rin	Θ
100	50	45°
150	50	54.75
200	50	60°
75	50	35.25°
50	50	0°

Table 7: Phase shift $\alpha + \Theta = 90$; LC Matching Network

Any phase shift greater than Θ results in X1 being a capacitor.

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A few closing statements before presenting the final design equations.

1. Three specific topologies when designing a π-matching network for impedance matching and a specific phase shift depending on the $\measuredangle\alpha$, the angle between V2 and Vout, and Θ, the desired phase shift.

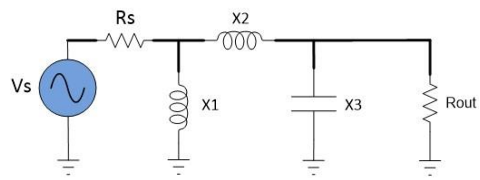

Figure 16: $\alpha + \Theta < 90$

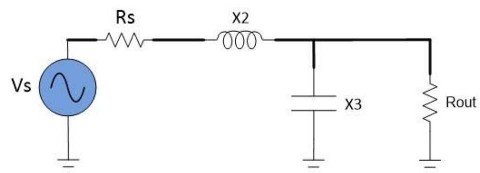

Figure 17: $\alpha + \Theta = 90$

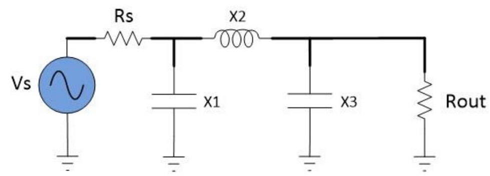

Figure 18: $\alpha + \Theta > 90$

2. This white paper design used a higher load resistance than the source. When the source resistance is higher than the load, the designer can still use these equations. But, the designer must reverse the source reactance, X1, and the load reactance, X3, in the **Figures 16 - 18**.

3. Analysis is with real resistance values for the source and load. Two approaches to handle reactive impedances. First, tune out the reactance, then use the **Table 8** design equations. This will result in using more than three elements. Second, use the phasor methods in this white paper to determine the reactance values.

4. This white paper calculated each reactance in a phase delay network. If the designer desires a phase advance network, then use the same phase delay reactance. But, use opposite reactance element and solve for the new element value using the appropriate reactance equation.

Vout = Rout	Iout = 1 A	(28)
$\text{Vin} = \sqrt{(\text{Rout} * \text{Rin})}$	$\text{Iin} = \text{Iout} * \sqrt{\frac{\text{Rout}}{\text{Rin}}}$	(29)
$\text{V2} = \sqrt{(\text{Rout} * \text{Rin}) + (\text{Rout})^2 - 2(\text{Rout})^{\frac{3}{2}} * (\text{Rin})^{\frac{1}{2}} * \cos(\Theta)}$		(30)
$\alpha = \cos^{-1}\left(\frac{(\text{Rout} * \text{Rin}) - (\text{Rout})^2 - (\text{V2})^2}{-2 * (\text{Rout}) * (\text{V2})}\right)$		(31)
$\Phi = 90° - \alpha$		(32)
$\text{I3} = \text{Iout} * \tan\Phi$		(33)
$\text{I2} = \frac{\text{Iout}}{\cos(\Phi)}$		(34)
$\text{I1} = \frac{\sin(\Phi - \Theta)}{\cos(\Phi)}$		(35)
$\text{X1} = +j\,\frac{\text{Vin}}{\text{I1}}$	$\alpha + \Theta < 90$	(36)
$\text{X1} = \infty$	$\alpha + \Theta = 90$	
$\text{X1} = -j\,\frac{\text{Vin}}{\text{I1}}$	$\alpha + \Theta > 90$	
$\text{X2} = +j\,\frac{\text{V2}}{\text{I2}}$		(37)
$\text{X3} = -j\,\frac{\text{Vout}}{\text{Iout}}$		(38)

Table 8: π-Network Design Equations